Calculation of Unsteady and Three-Dimensional Boundary-Layer Flows

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Theme

A NUMERICAL solution technique has been developed and analyzed for the laminar, time-dependent and three-dimensional boundary-layer equations. The method has been applied to a rotating flat plate in forward flight, which is of direct interest to helicopter rotors. The numerical method itself is a modification of the implicit schemes used in two- and three-dimensional boundary-layer problems and the most significant difficulties were the initial value problem and reversed flow. The combined problem of both time-dependent influences and three-dimensionality on the flat plate exhibited a much different behavior than either of the effects along. The most interesting behavior occurred near the retreating blade portion of the cycle where retreating blade stall has been a recurring problem.

Content

In the present paper, a three-dimensional and time-dependent boundary layer has been calculated by a numerical method. The flow problem studied consisted of a rotating flat plate in forward flight, and the flow conditions were laminar and incompressible. This particular problem presents many interesting features, both in the areas of numerical analysis and for understanding the complex interactions in three-dimensional and time-dependent flows.

The geometry of the present problem is shown in Fig. 1. During one part of the cycle, the forward flight velocity adds to the rotational velocity, while on the other part the two velocities subtract from each other. Also, it should be pointed out that the cross-flow velocity has components from both the forward and rotational motions.

A prerequisite for any boundary-layer problem is an inviscid solution to be used as an outer boundary condition for the viscous flow. The inviscid solution for the present problem has been worked out by McCroskey and Yaggy,[1] with various examples given by Dwyer and McCroskey.[2]

The appropriate laminar, incompressible boundary-layer equations for the present problem are:

continuity

$$(\partial u/\partial x)+(\partial v/\partial y)+(\partial w/\partial z) = 0 \qquad (1)$$

x-momentum

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2\Omega w = v\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\frac{\partial p}{\partial x} + \Omega^2 X \qquad (2)$$

z-momentum

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + 2\Omega U = v\frac{\partial^2 w}{\partial y^2} - \frac{1}{\rho}\frac{\partial p}{\partial z} + \Omega^2 Z \qquad (3)$$

where u, v, and w are the boundary-layer velocities in the x, y, and z directions, respectively; t is the time coordinate; v is the kinematic viscosity. The equations given above have some unusual properties which are not discussed in most textbooks on partial differential equations. For the x, z, and t coordinates initial value problems are posed, while the y coordinate poses a boundary value problem (the boundary conditions being the inviscid conditions and the wall no-slip conditions). Actually, both the x and z coordinates behave in a "timelike" manner, since no diffusion occurs in these directions. This timelike behavior can also be recognized by observing that the x and z derivative operators

$$u(\partial/\partial x) \text{ and } w(\partial/\partial y)$$

have the dimensions of time. Therefore, the system of equations given above has three timelike and one diffusion coordinates, and these are the only systems of equations of this type of which the author is aware. The physical consequences of the above analysis is that information is diffused along the y coordinate in the boundary layer and only convected along the other three timelike coordinates (this convection should not be taken literally).

Equations (1–3) are not the most efficient system for calculation purposes. By using coordinate transformations and a transformed system of equations, many problems associated with the boundary-layer equations can be alleviated.[2,4,5] The most important advantages contained in the transformed equations are: 1) removal of possible leading edge singularities, 2) the boundary-layer thickness in transformed coordinates is kept within well defined limits and grows very little, 3) methods of determining self consistent initial conditions are available, and 4) derivatives of the dependent variables do not change rapidly and large steps may be taken.

With the equations written in terms of the new coordinates, the initial conditions for the flat plate leading edge are easily determined and turn out to be a local Blasius type flow. However, the initial conditions for time and the crossflow coordinate pose more difficult problems. The problem of initial conditions for time can be overcome by taking advantage of the cyclic nature

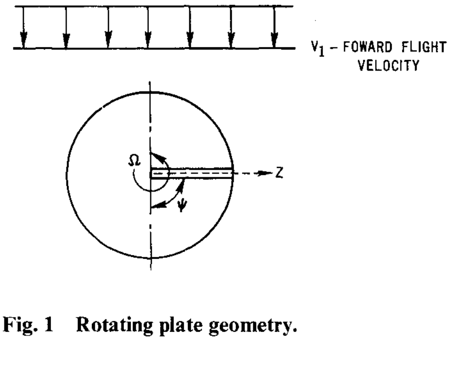

Fig. 1 Rotating plate geometry.

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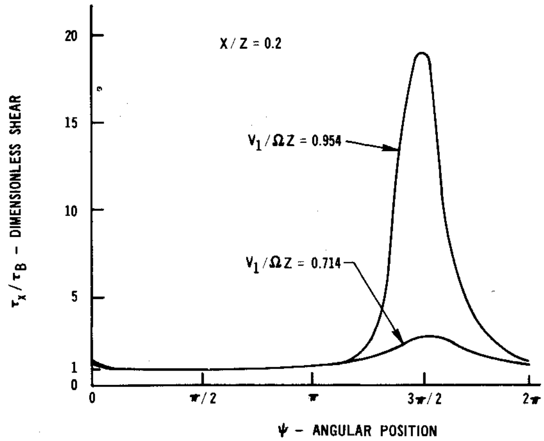

Fig. 2 Primary flow shear distribution.

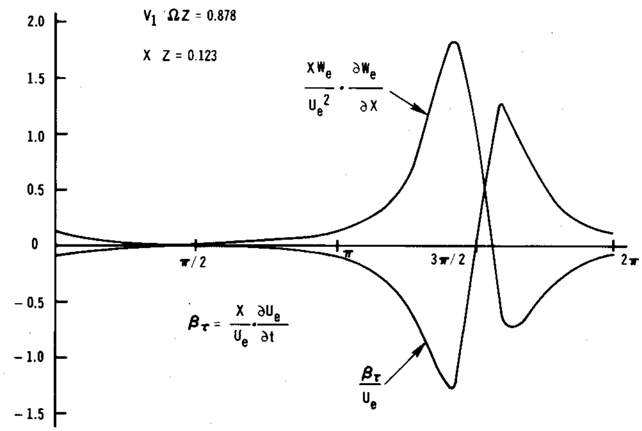

Fig. 3 Time variation of important pressure gradient terms.

of the problem. That is, a certain steady state solution is assumed to hold at $t = 0$ and then time-dependent calculations are carried out until they repeat themselves from cycle to cycle. Obviously, the choice of the steady state solution should be made carefully so that convergence will be obtained rapidly.[6] For the crossflow direction the same method of obtaining the initial conditions that was developed by Dwyer and McCroskey[2] was employed.

The numerical scheme used to integrate the equations can be classified as being implicit. The equations are evaluated at an unknown grid station and backward differences are taken for the t, x and z coordinates. The y-derivatives are evaluated at the unknown station in central difference form. The resulting set of simultaneous difference equations are solved by use of the Thomas algorithm since the unknowns appear in a tri-diagonal matrix form. This scheme is fast and very stable although the truncation error is only of order $|\Delta t|$ and $|\Delta\xi|$ for the backward differences. The results of our calculations will now be presented.

For the rotating flat plate in forward flight it was found that some regions of the flow were essentially the same as a two-dimensional Blasius flow, while in other regions there are significant departures due to time and rotational effects. The most interesting regime was where the retreating blade velocity was almost equal to the forward flight velocity. In this region, $\psi \simeq 270°$ (see Fig. 1), both the time-dependent and crossflow influences reach a maximum. Also, for the region very close to the hub the crossflow influences are always important for all values of ψ.

A typical example of the influence of time and rotation on primary flow wall shear is shown in Fig. 2, where the results have been given for various values of the significant parameters x/z (rotational influence parameter) and $V_1/\Omega Z$ (time-dependent influence parameter). The ordinate of the curve is the ratio of the primary wall shear to a local Blasius value and the abscissa is time. The basic reason that the effects are largest near $\psi \simeq 3\pi/2$ is that the primary flow velocity U_e goes through a minimum while the time and space dependent influences on pressure gradient are a maximum. The increase is due to a delicate and complicated balance, which can best be illustrated with the use of Fig. 3.

Figure 3 shows the interplay between the time and space dependent effects on the boundary-layer structure with a plot of the important driving pressure gradients. Plotted are the two most important terms which affect the x-direction pressure gradient, $xWe/Ue^2 \cdot \partial We/\partial x$ and $\beta\tau/Ue$. The first term is due to the crossflow derivative in the primary flow direction, while the second is due to unsteady influences. Both terms have a strong time dependence and exert their largest influence at the bottom of the boundary layer where the wall shear is determined. For $\pi \leqq \psi \leqq 3\pi/2$ the crossflow increased wall shear by overcoming the time-dependent effects. While for $3\pi/2 < \psi < 2\pi$ the time dependent term overcomes the crossflow one to increase the wall shear. The net effect is to uniformly increase wall shear, but for two different reasons. For very small values of x/z and $V_1/\Omega Z$, the above solutions agree qualitatively and quantitatively with the regular perturbation solutions of McCroskey and Yaggy.[1] A more complete summary of the effects of time and rotation, along with typical velocity profiles, can be found in Ref. 6.

References

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